

Exercise 57

Find an equation of the tangent to the curve at the given point.

$$y = 4 \sin^2 x, \quad (\pi/6, 1)$$

Solution

The aim is to find the slope of the tangent line at $x = \pi/6$. Take the derivative of y .

$$\begin{aligned} y' &= \frac{d}{dx}(4 \sin^2 x) \\ &= 4 \frac{d}{dx}(\sin x)^2 \\ &= 4(2)(\sin x)^1 \cdot \frac{d}{dx}(\sin x) \\ &= 4(2) \sin x \cdot (\cos x) \\ &= 4(2 \sin x \cos x) \\ &= 4 \sin 2x \end{aligned}$$

Plug in $x = \pi/6$.

$$y' \left(\frac{\pi}{6} \right) = 4 \sin \frac{\pi}{3} = 4 \left(\frac{\sqrt{3}}{2} \right) = 2\sqrt{3}$$

Use the point-slope formula with this slope and the given point $(\pi/6, 1)$ to get the equation of the tangent line.

$$y - 1 = 2\sqrt{3} \left(x - \frac{\pi}{6} \right)$$

$$y - 1 = 2\sqrt{3} x - \frac{\pi\sqrt{3}}{3}$$

$$y = 2\sqrt{3} x - \frac{\pi\sqrt{3}}{3} + 1$$

Below is a graph of the curve and its tangent line at $(\pi/6, 1)$.

